Bayesian Modelling and Sequential Learning of Latent COVID-19 Epidemic Dynamics

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Abstract

The global outbreak of the COVID-19 pandemic and its variants have severely affected the world health system and population. Sequential real-time disease surveillance models based on a Hidden Markov structure have played a prominent role in the evaluation and forecasting of epidemic infectious dynamics over time. In this work, we consider a novel Susceptible-Exposed (including Asymptomatic)-Infected- Removed (SE(A)IR) epidemic compartment model with a stochastic transmissions rate, in which we take into consideration the number of asymptomatic individuals that are still infectious. We also incorporate the effects of non-pharmaceutical interventions by adopting a mean-reverting Ornstein-Uhlenbeck process with embedded lockdown and vaccination factors for the transmission rate time-varying parameter. Bayesian inference is performed through the particle Markov chain Monte Carlo (p-MCMC) algorithm to simultaneously estimate parameters and latent epidemic status states.

Model Design

Covid-19 SEIR Compartment Model

$$\frac{dS_t}{dt} = -\beta_t(pE(A)_t + qI_t)\frac{S_t}{N}$$

$$\frac{dE(A)_t}{dt} = \beta_t(pE(A)_t + qI_t)\frac{S_t}{N} - (\alpha+\gamma)E(A)_t$$

$$\frac{dI_t}{dt} = \alpha E(A)_t - (\gamma + d)I_t$$

$$\frac{dR_t}{dt} = \gamma(E(A)_t + I_t) + I_t d$$

$$\frac{dx_t}{dt} = \theta(\mu_x - x_t) + \sigma\frac{dB_t}{dt}$$

$$\mu_x = b_0 + b_1 L_t + b_2 V_t^*$$

$$\beta_t = e^{x_t}$$

$$y_t \sim LogNormal(\alpha E(A) * r, \tau)$$

*: Vaccination effect incorporate for future model development

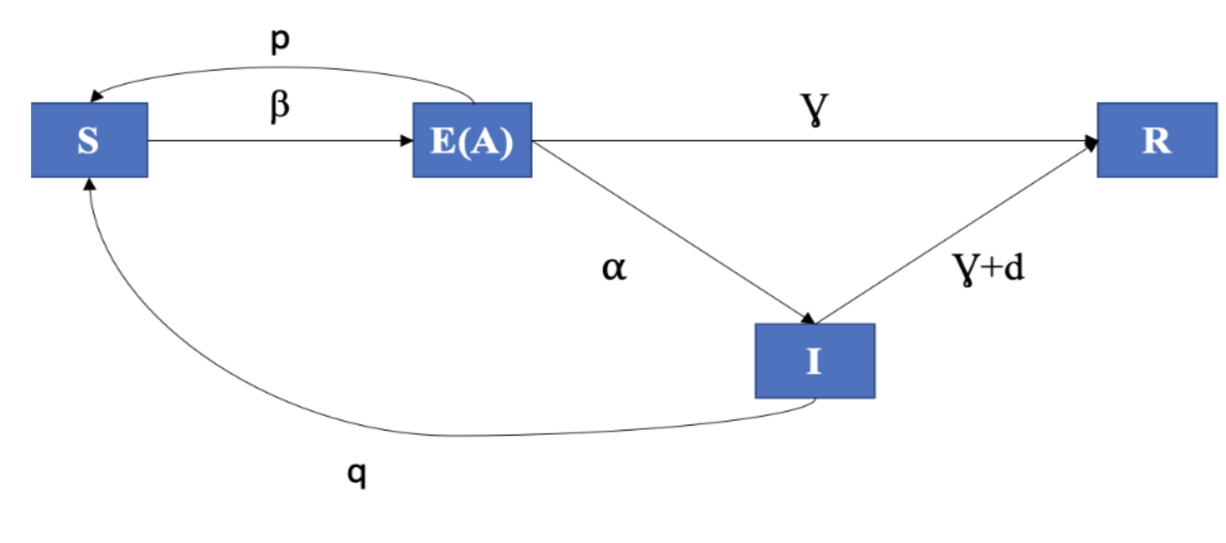


Figure 1. Infection chain of SE(A)IR compartment model

Parameter: Pre-setting parameters are: $(d,p,q,r)'$ whereas parameters to be estimated: $\Theta = (\alpha,\gamma,\theta,\tau,b_0,b_1,b_2)'$

(a) α: latency or onset symptoms development rate (where $\frac{1}{\alpha}$ is the mean time to onset of infectious)

(b) γ: recovered rate ($\frac{1}{\gamma}$ is the average time for an individual recover from the disease infectious system)

(c) θ: O-U process long-term mean reverting speed

(d) b_0, b_1 and b_2 are coefficients in O-U time varying drift μ_x

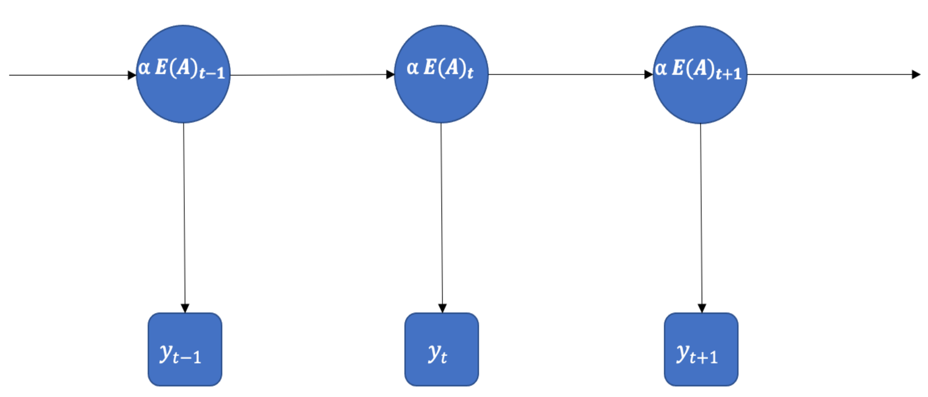
(e) τ: sd of observation distribution

(f) p: stands for the proportion of infectious counts (asymptomatic cases) in the compartment E(A); q: self-quarantine and hospital admitted symptomatic cases' virus spreading rate

(g) r: underreport rate

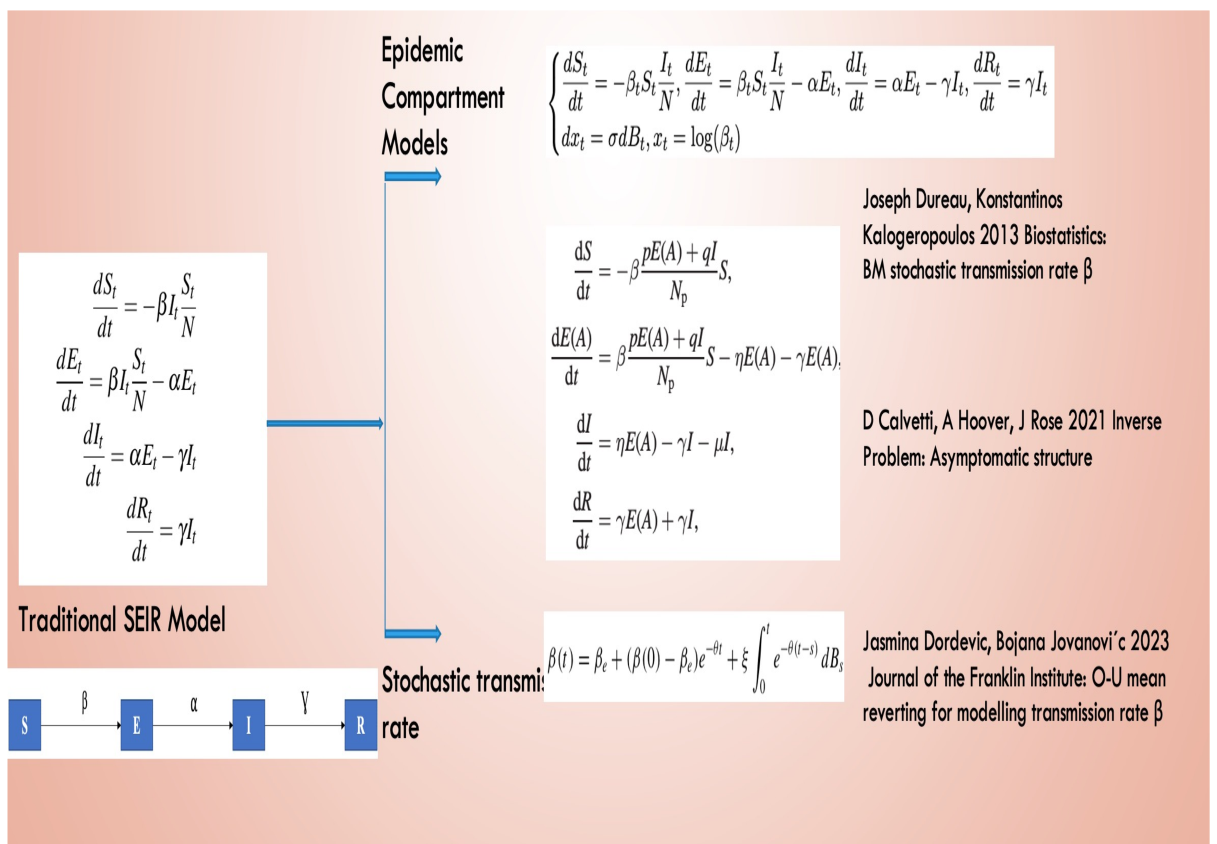
(h) d is the death rate

State Space Form



y: observations of new daily/weekly confirmed cases; αE(A): increasing symptomatic infections

Prior work & Contributions



Our work contribution: By adopting an asymptomatic case and exposed case combined structure, we take into consideration the number of COVID-19 asymptomatic individuals that are still infectious. We also incorporate the effects of non-pharmaceutical interventions by a mean-reverting Ornstein-Uhlenbeck process with embedded lockdown and vaccination factors for the transmission rate time-varying parameter. Bayesian inference of parameters and states will be conducted simultaneously using Particle-MCMC

State & Parameter Inference

Algorithm 1 Particle MCMC algorithm*

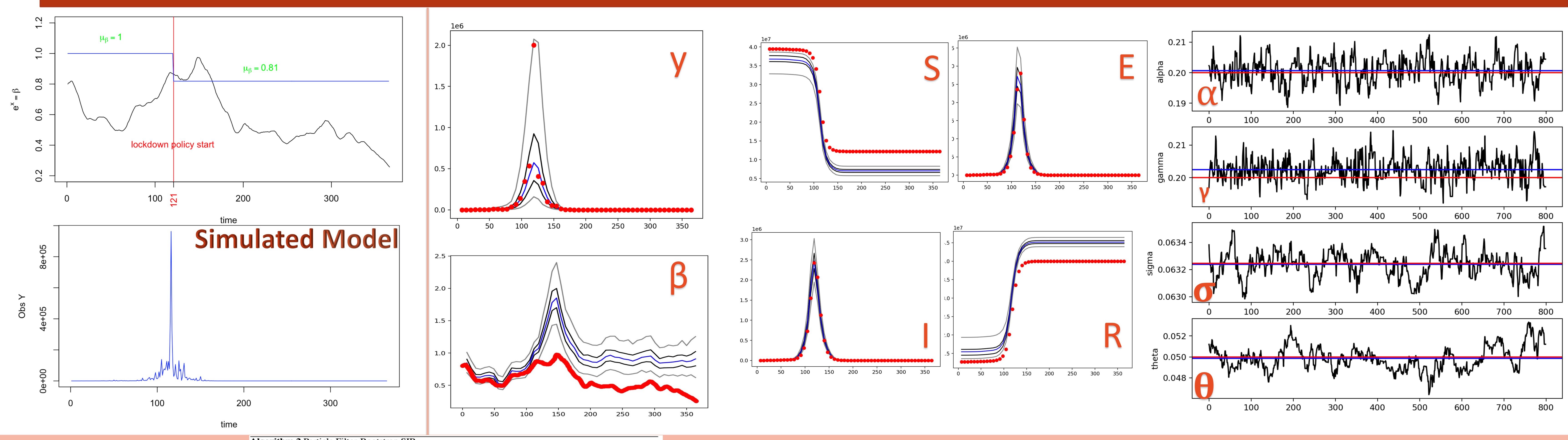
1. Initialize at iteration 0, j=0:
-Set $\Theta_{(0)}$ equals to prior means.
-Run Bootstrap particle filter with $\Theta = \Theta_{(0)}$, gain results: $\tilde{z}_{1:T(0)}$ and marginal likelihood estimate: $l_{\Theta_{(0)}}$
2. For iteration $i \geq 1$: Given the chain at $\Theta_{(i-1)}$ and $z_{1:T(i-1)}$:
-Sample $\Theta^* \sim q(\Theta^*|\Theta_{(i-1)})$
-Run Bootstrap particle filter targeting at $p_{\Theta^*}(z_{1:T}|y_{1:T})$, gain results: $\tilde{z}_{1:T(*)}$ and marginal likelihood estimate: $l_{\Theta^{(*)}}$
3. Accept $\Theta^{(*)}$ and $\tilde{z}_{1:T(*)}$ with probability:

$$\frac{l_{\Theta^{(*)}}p(\Theta^*)}{l_{\Theta_{(i-1)}}p(\Theta_{(i-1)})}\frac{q(\Theta_{(i-1)}|\Theta^*)}{q(\Theta^*|\Theta_{(i-1)})}$$

-Accept proposal values: then set $\Theta_{(i)} = \Theta^*$, $z_{1:T(i)} = z^*_{1:T}$ and $l_{\Theta_{(i)}} = l_{\Theta^*}$
-Otherwise, returns to $i-1$ th iteration values

* P-MCMC is implemented using LibBi-RBI package that developed by Pierre E. Jacob, Anthony Lee, Lawrence M. Murray, Sebastian Funk, Sam Abbott(2021)

Simulations & Results



Appendix: Bootstrap Particle Filter Algorithm

Algorithm 2 Particle Filter-Bootstrap SIR

States: $z_t = (S_t, E(A)_t, I_t, R_t, x_t)'$

1. Initialize at t=0: draw N samples from a presetting initial state density:

$$z_0^{(i)} \sim p(z_0)$$

for $i = 1,2,3,\cdots P$, and set initial importance weights as $\omega_0^{(i)} = \frac{1}{P}$

2. For time $t = 0,1,2,3\cdots T-1$, assume we already have resampled particles $\tilde{z}_t^{(i)}$ from the last time step (except from time step 0 to 1, we directly use $z_0^{(i)}$):

2.a Propagation step: Generate samples $z_{t+1}^{(i)}$ from the Bootstrap importance density:

$$z_{t+1}^{(i)} \sim p(z_{t+1}|\tilde{z}_t^{(i)}, \Theta)$$

for $i = 1,2,3,\cdots P$

2.b Filtering step: Calculate the importance weights for $i = 1,2,3,\cdots P$ at time $t+1$:

$$\omega_{t+1}^{(i)} \propto \omega_t^{(i)} p(y_{t+1}|z_{t+1}^{(i)}, \Theta)$$

As well as the normalized importance weights for $i = 1,2,3,\cdots P$:

$$\tilde{\omega}_{t+1}^{(i)} = \frac{\omega_{t+1}^{(i)}}{\sum_{j=1}^{P}\omega_{t+1}^{(j)}}$$

3. Resampling step: Resample the particles at $t+1$ generated at step 2.a according to the normalized weights as their respective probability masses:

$$\tilde{z}_{t+1}^{(i)} \sim \sum_{i=1}^{P}\tilde{\omega}_{t+1}^{(i)}\delta(z_{t+1} - z_{t+1}^{(i)})$$

After resampling, the weights now are uniformly: $\omega_{t+1}^{(i)} = \frac{1}{P}$ for $i = 1,2,3,\cdots P$.

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