Geometric ergodicity of Gibbs samplers for Bayesian error-in-variable regression

Austin Brown
Department or Statistics, University of Warwick, Coventry, UK
austin.d.brown@warwick.ac.uk

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Motivation and Goals

Problems in epidemiology and other sciences involve error in responses and covariates which classical linear regression does not take into account!

Main points:

- Provide **reliable** Gibbs samplers for Bayesian EIV regression.
- Estimators **always** satisfy a central limit theorem.
- Samplers are **robust** to misspecificiation of error distributions.

$$X = \mathcal{A} + E$$

Observed covariates   True covariates   Normal error

1) Model: Additional Error in Covariates

Error in response:

$$Y_i = \Theta^T Z_i + \mathcal{B}^T \mathcal{A}_i + \epsilon_i \qquad \epsilon_i \sim N(0, \Sigma)$$

Error in covariates:

$$X_i | \mathcal{A}_i \sim N(\mathcal{A}_i, V_i) \text{ (Classical)} \quad \text{or} \quad \mathcal{A}_i | X_i \sim N(X_i, V_i) \text{ (Berkson)}$$

1) Model: Error in Responses and Covariates

Error in response:

$$\mathcal{V}_i = \Theta^T Z_i + \mathcal{B}^T \mathcal{A}_i + \epsilon_i \qquad \epsilon_i \sim N(0, \Sigma)$$
$$Y_i | \mathcal{V}_i \sim N(\mathcal{V}_i, U_i)$$

Error in covariates:

$$X_i | \mathcal{A}_i \sim N_p(\mathcal{A}_i, V_i) \text{ (Classical)} \quad \text{or} \quad \mathcal{A}_i | X_i \sim N(X_i, V_i) \text{ (Berkson)}$$

1) Main Result

Bayesian priors:

Inverse-Wishart Σ Gaussian $(\Theta, \mathcal{B})$

Gaussian $\mathcal{A}_i$ (Classical) or $\mathcal{A}_i$ flat prior (Berkson)

> We can construct 3-variable Gibbs samplers for Bayesian EIV regression which are **always** geometrically ergodic!

- Reliably estimate posterior averages $\mathbb{E}(f)$ with the mean $\bar{f}_m$ from the Gibbs sampler.
- Gibbs samplers satisfy a central limit theorem:

$$\sqrt{m}\left(\bar{f}_m - \mathbb{E}(f)\right) \quad \textbf{converges to normal}$$

3) 3-Variable Gibbs Samplers

Algorithm 1: Gibbs sampler when errors in covariates

Generate Inverse-Wishart $\Sigma_t | \mathcal{A}_{t-1}, \Theta_{t-1}, \mathcal{B}_{t-1}$
Generate Gaussian $\Theta_t, \mathcal{B}_t | \mathcal{A}_{t-1}, \Sigma_t$
Generate Gaussian $\mathcal{A}_{i,t} | \Theta_t, \mathcal{B}_t, \Sigma_t$

Algorithm 2: Gibbs sampler when errors in respones and covariates

Generate Inverse-Wishart $\Sigma_t | \mathcal{A}_{t-1}, \mathcal{V}_{t-1}, \Theta_{t-1}, \mathcal{B}_{t-1}$
Generate Gaussian $\mathcal{V}_t, \Theta_t, \mathcal{B}_t | \mathcal{A}_{t-1}, \Sigma_t$
Generate Gaussian $\mathcal{A}_{i,t} | \mathcal{V}_t, \Theta_t, \mathcal{B}_t, \Sigma_t$

3) Limitations

- Generate articifial data from the Berkson error model
- The response m and the dimension of the covariates p are increasing in configurations $(m, p) = (1, 1), (2, 7), (3, 7)$.

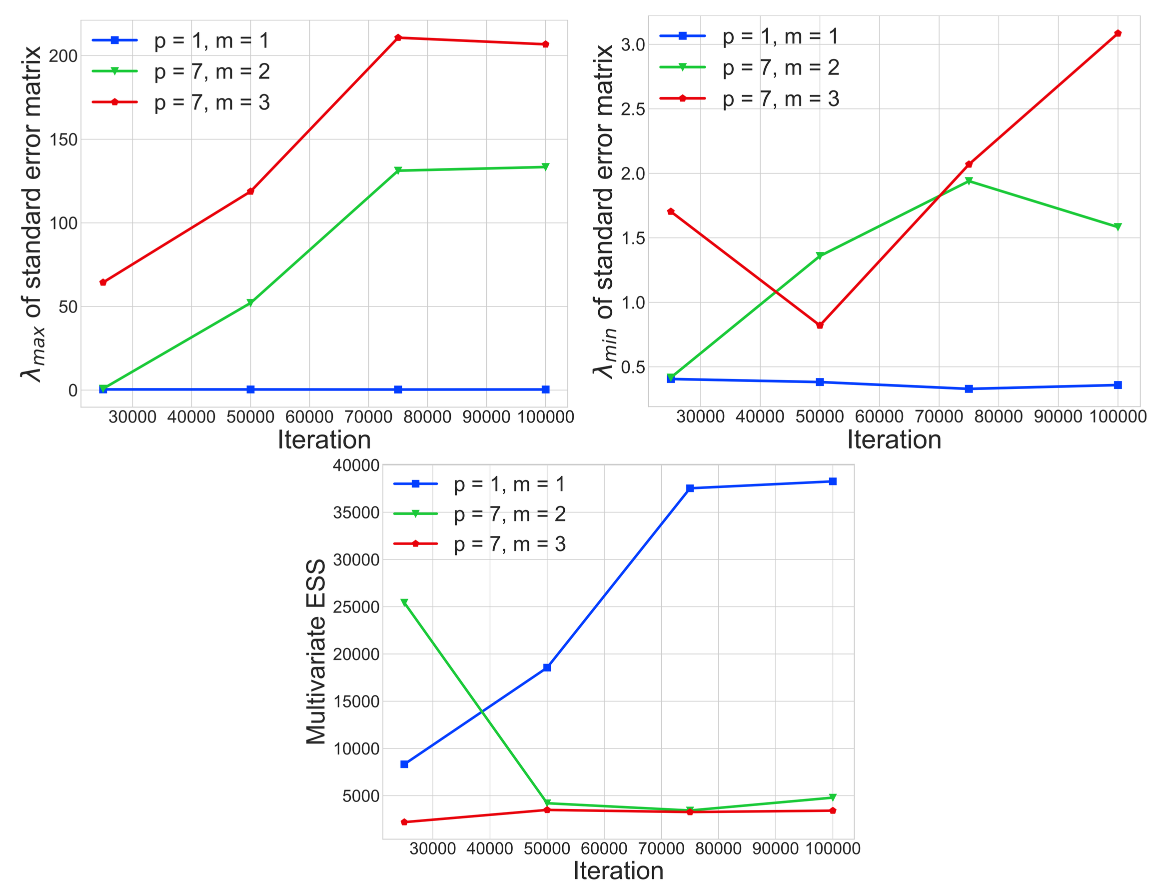

Figure 1: (a), (b) Largest and smallest eigenvalues of the CLT error matrix (c) Multivariate effective sample size

4) Robust to Error Misspecification

- Comparison of Berkson model with misspecified heavier tailed error in covariates: t-distribution with $df = 2$ and $df = 10$.

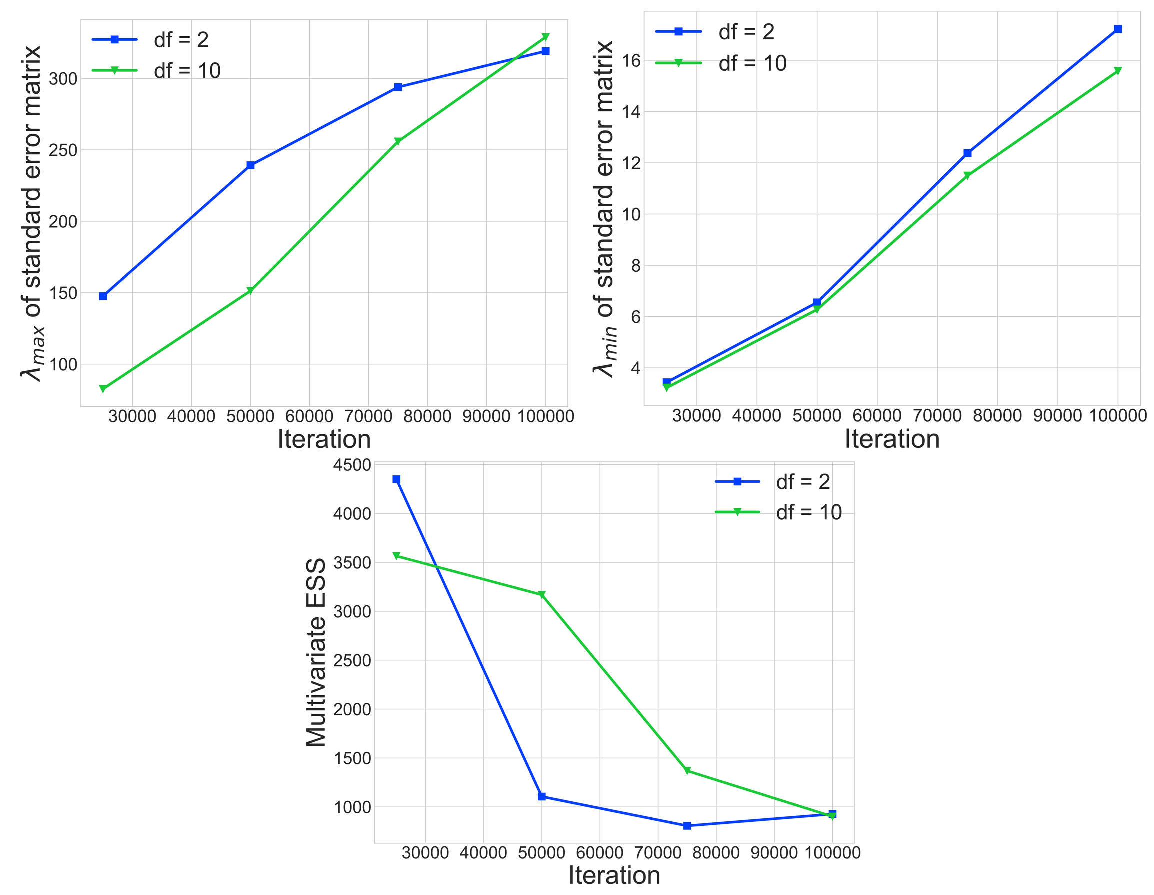

Figure 2: (a), (b) Largest and smallest eigenvalues of the CLT error matrix (c) Multivariate effective sample size

References and Acknowledgements

[1] Joseph Berkson. "Are There Two Regressions?" In: *J. Am. Stat. Assoc.* 45.250 (1950), pp. 164–180.

[2] Austin Brown. "Geometric Ergodicity of Gibbs Samplers for Bayesian Error-in-variable Regression". In: *preprint Arxiv:2209.08301* (2022).